An employer ran a year-long charity campaign. During the seventh week, $761 was donated. **SCORE: _____ / 5 PTS**

[a] If the amount donated every week was $42 higher than the amount donated the previous week, how much was donated over the entire year (52 weeks) ?

$a_7 = a_1 + 6d$

$761 = a_1 + 6(42)$ 1

$a_1 = 509$ $\frac{1}{2}$

$S_{52} = \frac{52}{2}(2(509) + 51(42))$ 1

$= 82,160$ $\frac{1}{2}$

[b] If the amount donated every week was 2.3% higher than the amount donated the previous week, how much was donated the first week ? **Round your answer to the nearest cent.**

$a_7 = a_1 r^6$

$761 = a_1 (1.023)^6$ 1

$a_1 = 663.94$ 1

Find $\binom{35}{7}$.

$$\frac{35!}{7!\,28!} = \frac{\cancel{35}\times 34\times \overset{11}{\cancel{33}}\times \overset{4}{\cancel{32}}\times 31\times \overset{5}{\cancel{30}}\times 29 \quad 1}{\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times 1 \quad 1}$$

$$= 34\times 11\times 4\times 31\times 5\times 29 \quad 1$$

ANY PRODUCT WHICH GIVES CORRECT FINAL ANSWER WITHOUT USING DIVISION OR !

$$= 6{,}724{,}520 \quad 1$$

SCORE: _____ / 4 PTS

Find the value of $\sum_{p=1}^{\infty} \frac{21}{2^{3p}}$. **HINT: Write out several terms of the series first.**

SCORE: _____ / 4 PTS

$$= \frac{21}{2^3} + \frac{21}{2^6} + \frac{21}{2^9} + \cdots = \frac{\boxed{\frac{21}{8}}}{\boxed{1 - \frac{1}{8}}} = \frac{\frac{21}{8}}{\frac{7}{8}} = \boxed{3}$$

$\times \frac{1}{8}$ $\times \frac{1}{8}$

1 2 1

Use the entries of Pascal's Triangle to expand and simplify $(3n^2 - 5n^7)^4$. 1 4 6 4 1 **SCORE: _____ / 5 PTS**

<u>You must show the intermediate step in the expansion to get full credit.</u>

$$(3n^2)^4 + 4(3n^2)^3(-5n^7) + 6(3n^2)^2(-5n^7)^2 + 4(3n^2)(-5n^7)^3 + (-5n^7)^4$$

$$= 81n^8 - 540n^{13} + 1350n^{18} - 1500n^{23} + 625n^{28}$$

$\frac{1}{2}$ POINT EACH

Using mathematical induction, prove that $1^2 - 2^2 + 3^2 - 4^2 + \cdots + (-1)^{n+1} n^2 = (-1)^{n+1} \frac{n(n+1)}{2}$ **SCORE: _____ / 12 PTS**

for all positive integers n.

BASIS STEP: $1^2 = 1 = (-1)^2 \frac{1(2)}{2}$

INDUCTIVE STEP:

ASSUME $1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k+1} k^2 = (-1)^{k+1} \frac{k(k+1)}{2}$

FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

$1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k+2}(k+1)^2$

$= 1^2 - 2^2 + 3^2 - 4^2 + \dots + (-1)^{k+1} k^2 + (-1)^{k+2}(k+1)^2$

$= (-1)^{k+1} \frac{k(k+1)}{2} + (-1)^{k+2}(k+1)^2$

$= (-1)^{k+1}(k+1)\left[\frac{k}{2} + (-1)(k+1)\right]$

$= (-1)^{k+1}(k+1)\left(\frac{k}{2} - k - 1\right)$

$= (-1)^{k+1} \frac{k+1}{2}(k - 2k - 2)$

$= (-1)^{k+1} \frac{k+1}{2}(-k-2)$

$= (-1)^{k+1} \frac{k+1}{2}\left[-(k+2)\right]$

$= (-1)^{k+2} \frac{(k+1)(k+2)}{2}$

BY M.I.

$1^2 - 2^2 + 3^2 - \dots + (-1)^{n+1} n^2$

$= (-1)^{n+1} \frac{n(n+1)}{2}$

FOR ALL POSITIVE INTEGERS